Manhãs de Matemática na AbERTA

#01. GEOMETRIA

21 de setembro de 2021
Auditório III, Palácio Ceia, Lisboa

https://videoconf-colibri.zoom.us/j/88141662805

A participação presencial é livre mas requer inscrição enviando email para: **fcosta@uab.pt**

HORA	**ORADOR**	**TÍTULO**
9h45–10h30	Antonio F. Costa (UNED)	On symmetries of knots
10h30–11h15	Carlos Florentino (FC-UL)	Geometry and topology of symmetric products
11h15–11h40	*COFFEE BREAK*	
11h40–12h25	Miguel Abreu (IST-UL)	Periodic orbits of Reeb flows on odd dimensional spheres
12h25–13h10	António Araújo (UAb)	Euclid, Taylor, and Spherical Perspective

ON SYMMETRIES OF KNOTS

Antonio F. Costa (Universidad Nacional de Educación a Distancia, Madrid)

Abstract: A knot is a closed and connected curve embedded in the three dimensional sphere S^3.

Since the origins of knot theory, one of the methods of studying knots is using the projections on a two dimensional sphere of S^3 (this is the case with the first publications on knot theory by P. G. Tait from 1877). A projection of a knot is its image by a central projection on a sphere (called projection sphere) so that no three points of the knot project on the same point. In a projection, at each double point, we will indicate, by a traditional graphical method, which segment "overpasses" and which "underpasses".

Alternating projections are a special type of projections that captured the attention of knot theorists from the very beginning; a knot projection is alternating if the crossings alternate under, over, under, over, and so on as one travels along it. A knot is alternating if it has an alternating projection.

For any geometrical (or even mathematical) object, it is natural to study its symmetries and this has also been one of the classical problems in knot theory. The simplest symmetries are certainly those given by a rotation around an axis of S^3. We will say that a knot K is q-periodic if there is a rotation of order q of S^3 that leaves K invariant (in the case $q = 2$, the axis of the rotation must not intersect the knot).

The easiest way to find symmetries of a knot is on its projections. A projection is q-periodic if there is a rotation of order q of the projection sphere leaving the projection of the knot invariant with its overpasses and underpasses. Determining whether a projection of a knot K is q-periodic is something that can be done, but the main problem is to determine in which of the many infinite projections of K the symmetries are to be studied. The following main theorem implies that for alternating knots, the possible q-periodicities with $q > 2$ can be "visualized" on alternating projections.

Visibility Theorem *Let K be a prime alternating knot that is q-periodic with $q \geq 3$. Then K has a q-periodic alternating projection.*

Note that the condition $q > 2$ is clearly necessary. The simplest example of alternating 2-periodic knot without 2-periodic alternating projections is the trefoil knot.

We will talk about other other types of symmetries of knots that are well studied. One of the most popular symmetries is the achirality. A knot is achiral if there is a symmetry of the knot which is an orientation reversing homeomorphism of S^3.

Finally we will present some applications.

GEOMETRY AND TOPOLOGY OF SYMMETRIC PRODUCTS

CARLOS FLORENTINO (Faculdade de Ciências, Universidade de Lisboa)

Abstract: Given $n \in \mathbb{N}$, the nth *symmetric product* of a topological space X is the space of unordered n-tuples of points of X, ie, the quotient $\mathsf{Sym}^n X = X^n/S_n$ of the n-fold Cartesian product of X under the symmetric group S_n. A beautiful formula of Macdonald (from 1962), for the Poincaré polynomial of symmetric products, shows that a lot of information on the topology of $\mathsf{Sym}^n X$ can be obtained from the one of X. Additionally, if X is a manifold, or a Lie group, or an algebraic variety, etc, then $\mathsf{Sym}^n X$ still carries much of the structure of X, and very explicit descriptions can be given in many cases. Even the simplest spaces $X = \mathbb{R}, \mathbb{C}$, a general vector space or $X = S^1$ or S^2 provide interesting examples, along with surprising relations with algebra, geometry, and representation theory.

In this talk, after considering basic examples of $\mathsf{Sym}^n X$, we will focus attention on compact Riemann surfaces, Lie groups, and smooth algebraic surfaces (two dimensional complex manifolds). We end with two further surprises: a new formula for the Poincaré polynomials and Euler characteristics of commuting elements in Lie groups; and the cases of $X = (\mathbb{C}^*)^2$ and of the cotangent bundle of a compact Riemann surface Σ, where $\mathsf{Sym}^n X$ appears as hyper-holomorphic subvariety in the moduli space of Higgs bundles of rank n over Σ, and is actually the full moduli space, when Σ is an elliptic curve.

PERIODIC ORBITS OF REEB FLOWS ON ODD DIMENSIONAL SPHERES

MIGUEL ABREU (Instituto Superior Técnico, Universidade de Lisboa)

Abstract: An embedding of the 2n+1 dimensional sphere in the real 2n+2 dimensional vector space gives rise to a 1-parameter autonomous flow on the sphere, called the characteristic flow. If the image of the sphere by the embedding bounds a starshaped domain, the corresponding characteristic flow is a Reeb flow. A long

standing and important conjecture, which is still very much open, states that any of these Reeb flows on the 2n+1 dimensional sphere has at least n+1 geometrically distinct periodic orbits. In this talk I will present illustrative examples and some results motivated by this conjecture in the convex case, including recent joint results with Leonardo Macarini obtained using index theory of Long and Floer homology.

EUCLID, TAYLOR, AND SPHERICAL PERSPECTIVE

António Araújo (Universidade Aberta, Lisboa)

Abstract: Is perspective a proper mathematical object? If it is, what is it? We will look at perspective from a slightly unusual point of view, starting by trying to understand what Euclid meant with his book. No, not *The Elements,* the other one: the *Optics.* I claim that what he worked on is properly seen as (spherical) anamorphosis, and that the later codification of linear perspective was both an advance and a retrocess, losing Euclid's generality but opening the way to Taylor's main invention: no, not the series, the other one: vanishing points.