Manhãs de Matemática na AbERTA

#03. SISTEMAS DINÂMICOS

6 de julho de 2023
Delegação Regional do Porto, Universidade Aberta
Rua do Amial 762 - Porto

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A participação presencial é livre mas requer inscrição enviando email para:
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HORA	ORADOR	TÍTULO
9h45–10h30	Fernando Pestana da Costa (UAb)	The dynamics of a coagulation type model of silicosis
10h30–11h15	João Teixeira Pinto (IST-UL)	Bifurcations in liquid crystals models
11h15–11h40	*COFFEE BREAK*	
11h40–12h25	Maria Joana Torres (UM)	Generic behaviours of dynamical systems
12h25–13h10	Alexandre Rodrigues (ISEG-UL)	Bifurcações em sistemas epidemiológicos

The dynamics of a coagulation type model of silicosis

Fernando Pestana da Costa (UAb e CAMGSD, IST, Univ. Lisboa)

Abstract: We present a system with a countable number of ordinary differential equations of coagulation type that can be considered a simple model for silicosis. Silicosis is a respiratory disease due to the ingestion of quartz dust and their acumulation in the lungs. The mathematical model consists in an ODE for the concentration of free quartz particles, an ODE for the concentration of macrophages without quartz (cells of the immune system that identify, capture and try to expel entities strange to the body), and a countable number of ODEs each one describing the concentration of macrophages with a number $i \in \mathbb{N}$ of captured quartz particles. We briefly describe basic results such as existence, uniqueness, regularity, and semigroup results of the set of solutions. Then we study the dynamics of the infinite dimensional system in the case of a particular class of rate coefficients that allows for the decoupling of the full infinite dimensional system into a finite dimensional one and a lower triangular infinite system. By the analysis of the finite dimensional subsystem we conclude that it has a saddle-node bifurcation, possessing two equilibria when a bifurcation coefficient which

is the ratio of the input rate of quartz to the rate of creation of empty macrophages is below a critical value, and no equilibria above that value. Then we study the stability properties of the bifurcating branches and prove that the stable branch corresponds to equilibria of the full infinite dimensional system which are locally exponentially asymptotically stable in the strong topology of the phase space. We end with some comments about a possible interpretation of these results.
The presentation is based on joint works with P. Antunes, M. Drmota, M. Grinfeld, J.T. Pinto, and R. Sasportes.

Bifurcations in liquid crystals models

João Teixeira Pinto (CAMGSD, IST, Univ. Lisboa)

Abstract: Liquid crystals are intermediate phases between the best known solid and liquid ones in the sense that, like liquids they flow but like solids their molecules tend to arrange themselves in ordered ways both in position and in orientation. These materials are ubiquitous in nature but also in everyday life applications, the most obvious ones are the Liquid Crystals Devices (LCD) displays and monitors. The way these devices work are essentially based upon the orientational geometry of the liquid crystal molecules inside a LCD cell and the way it reacts to the application of external electromagnetic fields. These devices explore the fact that different orientational geometries correspond to different optical properties of the material. The transition between two different geometric arrangements corresponding to different values of the applied external fields, known in liquid crystals modelling as the Freedericksz transition, correspond mathematically to a bifurcation problem that can be studied by techniques of the qualitative theory of differential equations. Some of these will be the subject of this talk where it will be presented some recent results obtained in joint work with F. P. da Costa, E. C. Gartland Jr., M. Grinfeld, M. I. Méndez, N. J. Mottram, e K. Xayxanadasy. Also some work in progress on the new field of active liquid crystals will be referred.

Generic behaviours of dynamical systems

Maria Joana Torres (CMAT, Univ. Minho)

Abstract: Dynamical systems is the study of the long-term behaviour of evolving systems. The foundations were set by the master work of Henri Poincaré, *Les Méthodes Nouvelles de la Mécanique Céleste* $(1892-1899)$, with fundamental questions concerning the stability of the solar system. Attempts to answer those questions revealed the incapacity of solving *exactly* the mathematical questions arising from physical systems. It appeared that understanding *typical* systems, or systems *in general*, was mathematical more fruitful.

The aim of the modern theory of dynamical systems is thus to describe the behaviour of *typical* (or *generic*) trajectories, for *typical* evolution laws. Furthermore, when dealing with real-world systems, neither the initial data neither the evolution law are known exactly. For these reasons, one is most interested in properties that are *stable*, i.e., that are persistent by small perturbations of the evolution law.

In this talk it will be presented an overview and recent results on stability and generic behaviours of dynamical systems.

Bifurcações em sistemas epidemiológicos

Alexandre Rodrigues (ISEG, Univ. Lisboa e CMUP, Univ. Porto)

Abstract: Nesta sessão, vamos descrever o modelo epidemilógico SIR (modificado) com e sem vacinação com a particularidade de que o parâmetro que modela a taxa de infeção apresenta variações sazonais. A digressão assentará na descrição de conceitos matemáticos ligados a equações diferenciais com perturbações periódicas, teoria da bifurcação, teoria do caos e equações impulsivas. Faremos ainda uma ponte entre os resultados matemáticos obtidos e descrições empíricas já existentes na literatura. Os conteúdos da palestra resultam de trabalho conjunto com João Maurício de Carvalho (Un. Porto).

[1] J. M. Carvalho, A. Rodrigues, SIR Model with vaccination: Bifurcation analysis, Qualitative Theory of Dynamical Systems, 2023

[Organização de: F. Costa (UAb & CAMGSD-IST-UL) e M. Bessa (UAb & CMUP-UP)]